Efficiency of Structure-Control Systems

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The concept of efficiency is introduced as a nondimensional measure of structure-control system evaluation. This concept has the potential to characterize quantitatively and qualitatively the control designer's ability in dealing with some of the problem areas in structural control. The efficiency of the system is defined as the ratio of two control power functionals pertinent to the structure-control problem where each functional represents an average control power consumed during the control period. The efficiencies are given both for partial differential and finite-element model equations of motion of the distributed parameter structure (DPS). It is shown that the behavior of the entire full-order DPS can be ascertained based on the quantities computed from the control design model alone without any knowledge of the uncontrolled dynamics of the system. The study is based on the premise that, within the given constraints, structure-control systems, which are most efficient, must be considered for implementation. Specifically, analysis of control systems from an efficiency point of view is addressed, and an efficient model reduction approach is proposed. The concepts are illustrated by studying the efficiency of various linear quadratic regulator solutions for the ACOSS-4 structure.

I. Introduction

THE purpose of this paper is to study a nondimensional measure of structure-control system performance that has the potential to characterize both quantitatively and qualitatively the designer's ability in dealing with some of the problem areas such as assessment of control spillover effects, model and controller order reduction, input configuration, and the interaction between structural and control variables from the structure-control system point of view. The nondimensional measure is defined as the efficiency of the system.

The concept of efficiency is widely encountered in thermal and thermomechanical sciences. In these fields an efficiency is defined as a nondimensional ratio of two scalars that represent energy or energy-related quantities. Typically, one of the scalars characterizes a theoretically ideal but physically irrealizable process, and the other characterizes an actual physically realized process. The difference between these two scalars represents waste of the total available energy or energy-related quantity in realizing the actual physical process and is regarded as an irreversibility inherent in the physical system. Many different forms of efficiency are defined in thermomechanical sciences depending on how the theoretically ideal and the physically realized actual processes are identified in a certain application. Some examples are propulsion efficiency, heat engine and heat pump efficiency, adiabatic compressor efficiency, brake efficiency, overall efficiency, etc.[9] In thermomechanical disciplines an important objective is to design a system with high or maximum efficiency consistent with the physical constraints. In practical engineering terms, the thermomechanical systems are desired that use (or convert) the largest fraction of the total available energy in realizing the physical function of that system, thus causing the minimum waste of resources.

The utility of efficiency concepts as analysis and design tools in thermomechanical systems is well established and provides valuable physical insight to the working of the system. The question arises regarding whether such a time-tested concept in thermomechanical discipline can be extended to the field of distributed parameter structure-control systems and yield comparable practical value and physical insight for the analysis and design of such systems. This paper is a quest in that direction and presents a conceptual framework to establish the usefulness of efficiency concept for structure-control systems.

For structure-control systems, the efficiency concept is defined as the ratio of two control cost functionals pertaining to the particular structure-control design. The control cost functionals are judiciously defined to represent the average control powers consumed during the control period. For the purpose of defining efficiencies, four control power functionals are relevant. Out of these four control powers a relative model efficiency and a global efficiency can be defined for a structure-control system with maximum possible percent efficiency of 100 for each. The two efficiencies are related by a modal efficiency coefficient.

Relative model efficiency is defined as the ratio of a modal control power functional to a real control cost functional associated with any control design model. Physically, relative model efficiency represents the fraction of real control power expended on the DPS usefully absorbed by the control design model. The relative model efficiency can be defined for all types of control-configurations on the DPS, that is, whether spatially discontinuous (discrete) or spatially continuous controls are used to achieve control. Predominantly, it indicates the effect of finite dimensionalization of the control design model in using the available control power and configuration.

On the other hand, global efficiency is defined as the ratio of a globally minimum real control power based on a spatially continuous input profile to a real control power to achieve the same control objectives with a spatially discontinuous input profile. The definition is based on dynamically similar control

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systems.[1] Physically, the global efficiency is related to the degree of extra control power associated with a spatially discontinuous input configuration in controlling a DPS vs accomplishing the same task with a spatially continuous input profile. Hence, the global efficiency predominantly indicates the effect of input configuration of a given control design model.

Modal efficiency coefficient is a proportionality constant between the global efficiency and the relative model efficiency. Thus, with these quantities, the effect of finite dimensionalization of the control design model and the effect of nature of spatial discretization of the input profile in controlling a DPS can be studied.

The concept of efficiency is introduced to address particular issues of DPS control. A good physical understanding of the concept is best brought about by considering a DPS formulation. Therefore, the basic aspects of the concept are presented from the perspective of a DPS formulation. On the other hand, in practice, for a complex structure only a spatially discrete formulation is explicitly available. Since such a spatially discrete formulation acts as a surrogate for DPS formulation, the transition of the definitions of efficiency to spatially-discrete structural dynamics is also presented. In particular, since spatially discrete models are almost always obtained via the finite-element method (FEM), specific interpretations of the concepts for the finite-element models are given.

The premise of the concepts presented in this paper is that a structure-control system must strive to achieve the highest possible efficiencies; i.e., the control system should be designed such that as much of the real control power as possible should be channeled to the reduced-order control design model, leaving little or, if possible, no control power spillover for the truncated dynamics. This objective requires maximization of the relative model efficiency. Furthermore, any nonoptimal control design, characterized by a spatially discontinuous input profile, should try to approximate the performance of the globally optimal solution characterized by a dynamically similar spatially continuous input profile. This objective requires maximization of the global efficiency. Based on the proposed concepts, an efficient model reduction approach can be proposed in that for a given input configuration the components that contribute least to the efficiencies should be discarded.

The efficiencies are functions of control parameters such as the number of control inputs and locations, method of control, and model order, in addition to structural parameters such as natural frequencies and mode shapes. All relevant parameters can be altered to change the efficiency of the hybrid structure-control system. An important aspect of the results is that the performance of the entire infinite-dimensional (∞-D) DPS can be studied based on quantities computed from the finite-dimensional control design model alone without any knowledge of the truncated dynamics. The concepts introduced are valid irrespective of the theory or method by which the controls are designed. The efficiency concept can be used both for analysis and design of the structure-control system. In this paper, the emphasis is on the analysis of the structure-control system. The utility of the concepts are demonstrated to evaluate various control designs for the ACOSS-4 structure by studying their efficiencies. Because the linear quadratic regulator (LQR) theory is the most widely used approach for control, the illustrations on the ACOSS-4 are based on the LQR designs.

II. Control Objectives for the Distributed Parameter Structure

We consider the DPS equation of motion

$$m(p)\ddot{u}(p,t) + \mathcal{L}[u(p,t)] = f(p,t) \tag{1}$$

where m, u, $\mathcal{L}$, and f are mass distribution, displacement field vector, stiffness operator matrix, and the input field vector, respectively, and p denotes a position vector, which will be suppressed in the following for convenience. In general, Eq. (1) will represent the three-dimensional partial differential equations of motion. For convenience, we shall assume $\mathcal{L} > 0$.

The eigenvalue problem of Eq. (1) is

$$\omega_r^2 m \phi_r = \mathcal{L}\phi_r \tag{2}$$

with the orthogonality relations

$$\int_{D(p)} \phi_r^T m \phi_s \, dD(p) = \delta_{rs}, \qquad \int \phi_r^T \mathcal{L} \phi_s \, dD(p) = \omega_r^2 \delta_{rs} \qquad r,s = 1, 2, \ldots \tag{3}$$

where ω_r is the natural frequency, ϕ_r is the corresponding vector of eigenfunctions, and $D(p)$ is the structural domain. Introducing the modal expansion

$$u = \sum_r \phi_r(p)\xi_r(t) \tag{4}$$

Equation (1) is transformed to

$$\ddot{\xi}_r(t) + \omega_r^2 \xi_r(t) = f_r(t), \qquad r = 1, 2, \ldots \tag{5}$$

in the modal configuration space, where ξ_r is a modal configuration coordinate and $f_r(t)$ represents a modal input coordinate. Similar to modal expansion (4) of the displacement field vector, a modal expansion for the input field vector can be written as

$$f(p,t) = \sum_r m(p)\phi_r(p)f_r(t) \tag{6}$$

From Eqs. (3) and (6) the modal input coordinate f_r can be obtained as

$$f_r(t) = \int \phi_r^T(p) f(p,t) \, dD \tag{7}$$

The control objective on the ∞-D DPS is to insure proper allocation of n pairs of eigenvalues of a set of $\{n\}$ modes from Eq. (5). Another objective is to minimize control spillover effects so that the response of the control design will not be degraded by excessive control spillover. In improving stability characteristics of time-invariant linear systems, the premise of all control methods is to obtain desirable eigenvalue locations either directly or indirectly. The phrase "eigenvalue allocation" is used here in a general sense to address both direct and indirect means of allocation. As examples, a direct eigenvalue allocation technique is the Simon-Mitter algorithm or any other technique in which desired eigenvalue locations are imposed as explicit constraints in obtaining the control gains; on the other hand, an indirect eigenvalue allocation technique is to use the LQR theory, by which eigenvalue positions are obtained indirectly as a byproduct of optimizing a performance measure. Hence, the LQR approach is sometimes categorized as an optimal eigenvalue allocation technique. No implication is intended in this paper regarding the use or necessity of direct eigenvalue allocation techniques in understanding and applying the proposed concepts of efficiency. In the sequel, the emphasis will be on the generic meaning of the phrase "eigenvalue allocation" as a qualifying phrase for the function of the control system, which may have been designed by either a direct or indirect allocation, whatever the case may be.

From the control objectives stated earlier, one would infer the ideal control to be the one by which $\{n\}$ pairs of closed-loop eigenvalues are located as desired with the minimum control power, and control spillover is eliminated completely. The solution for such an ideal control for the ∞-D DPS [Eq. (1)] will be stated in Sec. IV.

III. Global Control Power for the DPS

Whatever one's favorite control design may be to satisfy the control objectives, one can define and compute a global control evaluation functional for the ∞-D DPS in the form

$$S = \int_0^\infty \int_{D(p)} m^{-1}(p) f^T(p,t) f(p,t)\, dD(p)\, dt \tag{8}$$

Equation (8) represents the total quadratic control cost expended on the actual DPS. The control cost [Eq. (8)] is dimensionally the average power consumed by the control design over the control period.[10,11] Because Eq. (8) is the control power on the entire DPS, it is recognized as a global quantity. Since the global control power defined by Eq. (8) is computed from the physical input field vector, it will also be referred to as the real control power S^R, where superscript R denotes "real"; hence,

$$S^R = \iint m^{-1}(p) f^T(p,t) f(p,t)\, dD\, dt \tag{9}$$

S^R can be computed for any given $f(p,t)$ regardless of the details of the control design technique by which it is computed. By a similar motivation, we define a global modal control power functional as

$$S_\infty^M = \sum_{r=1}^{\infty} \int_0^\infty f_r^2(t)\, dt \tag{10}$$

where M denotes that the quadratic controls are the modal control input coordinates, and the subscript ∞ implies that all modes, hence the entire DPS, are considered in the computation. In contrast to S^R, which involves the real input field, S^M seems to represent an abstract quantity, since modal inputs are used in its definition. However, S^R and S_∞^M are related.

Substituting the modal expansion (7) into definition (9) and using the orthogonality relations, one obtains

$$S^R = \iint m^{-1} \sum_{r=1}^{\infty} m\phi_r f_r \sum_{s=1}^{\infty} m\phi_s f_s\, dD\, dt = \sum_{r=1}^{\infty} \int f_r^2(t)\, dt \tag{11}$$

Hence,

$$S^R = S_\infty^M \tag{12}$$

Identity (12) defines an invariance property. The global control power for the DPS is frame indifferent; it is the same whether one studies it in the real space of Eq. (9) or the modal space of Eq. (10).

The global modal control power S_∞^M can be decomposed into

$$S_\infty^M = \sum_{r=1}^{n} \int_0^\infty f_r^2(t)\, dt + \sum_{r=n+1}^{\infty} \int_0^\infty f_r^2(t)\, dt \tag{13}$$

$$S_\infty^M = S_C^M + S_U^M \tag{14}$$

and from Eq. (12),

$$S^R = S_C^M + S_U^M \tag{15}$$

where the definitions of S_C^M and S_U^M should be evident from Eqs. (13) and (14). S_C^M is the portion of the control power S^R channeled for control of the $\{n\}$ modes, and S_U^M is the remaining control power channeled into, or better said, spilled over to the uncontrolled modes. If the set of modes $\{n\}$ is referred to as the control design model, S_C^M will be termed "design model control power." Similarly, S_U^M will be referred to as "control power spillover."

Since S^R, S_C^M, S_U^M are positive definite quantities,

$$S^R \geq S_C^M \tag{16}$$

where the equality is satisfied if and only if $S_U^M = 0$, that is, when there is no control power spillover.

An important feature of the control of the ∞-D DPS is imbedded in inequality (16). Because S^R is computed by using the real $f(p,t)$ applied to the actual structure, and S_C^M is computed by using the modal inputs to the finite-dimensional control design model, inequality (16) relates how the control design model performance stands relative to the actual DPS. It is clear that any mismatch between S^R and S_C^M would automatically mean that some of the control power is lost to the residual modes, $S_U^M \neq 0$. On the other hand, in accordance with the control objectives, the ideal control system for the DPS would yield $S_U^M = 0$; that is, it would minimize the spillover power.

It remains to address the specifics of how one might realize a minimum global control power to achieve the control objectives. To this end, we shall assume linear state-feedback control.

IV. Globally Optimal Control for the DPS

In the absence of other objectives and design constraints, it is reasonable to try to achieve the control objectives stated in Sec. II with the minimum amount of global control power. Hence one can state the optimization problem as the following:

Minimize S_∞^M, or equivalently, S^R subject to

$$\rho\{\pm i\omega_r\} \to \bar{\rho}\{\bar{\alpha}_r \pm i\bar{\beta}_r\}, \qquad r = 1, 2, \ldots, \infty \tag{17}$$

This is an optimization problem for the ∞-D DPS. The uncontrolled eigenvalues $\pm i\omega_r$ are relocated to specified positions $\bar{\alpha}_r \pm i\bar{\beta}_r$, where $\rho\{\ \}$ represents an eigenvalue spectrum. There are no restrictions on the constraint values $\bar{\alpha}_r$ and $\bar{\beta}_r$. The case where only n pairs of eigenvalues are relocated to new positions and the remaining residual pairs are not moved is a special case of the preceding formulation, since we can always write

$$\rho\{\pm i\omega_r\} \to \bar{\rho}\{\bar{\alpha}_r \pm i\bar{\beta}_r\}, \qquad r = 1, 2, \ldots, n \tag{18a}$$

$$\rho\{\pm i\omega_r\} \to \bar{\rho}\{\pm i\omega_r\}, \qquad r = n+1, \ldots, \infty \tag{18b}$$

The solution of the minimum global control power is known to be[1,2]

$$f_r^*(t) = \{\omega_r g_{r1}^* \xi_r^*(t) + g_{r2}^* \dot{\xi}_r^*(t)\} \tag{19}$$

$$\omega_r g_{r1}^* = \omega_r^2 - (\bar{\alpha}_r^2 + \bar{\beta}_r^2), \qquad g_{r2}^* = 2\bar{\alpha}_r \tag{20}$$

where (*) denotes the optimum quantities, and $r = 1, 2, \ldots, \infty$ for both equations. Substituting the solution (19) into Eq. (6) one obtains the optimum input field vector, which can achieve the desired eigenvalue locations with minimum control power[3]:

$$f^*(p,t) = \int \{G_1^*(p,p')u(p',t) + G_2^*(p,p')\dot{u}(p',t)\}\, dD(p') \tag{21}$$

where G_1^* and G_2^* are identified as symmetric optimum distributed control influence (Kernel) functions:

$$G_1^*(p,p') = \sum_{r=1}^{\infty} \omega_r g_{r1}^* m(p)\phi_r(p)\phi_r(p')m(p') = G_1^*(p',p) \tag{22a}$$

$$G_2^*(p,p') = \sum_{r=1}^{\infty} g_{r2}^* m(p)\phi_r(p)\phi_r(p')m(p') = G_2^*(p',p) \tag{22b}$$

We observe the following characteristics of the optimal control solution [Eq. (21)]: The optimal modal input coordinate f_r^* is a feedback of only the corresponding rth modal coordinate; therefore, optimal modal control coordinates are independent of each other. This feature of feedback control has come to be known as independent modal space control.[4] The corresponding optimal input field $f^*(p,t)$ is spatially continuous since the modal synthesis of spatially continuous functions $m(p)\phi_r(p)$ is a spatially continuous function. Without proof, we also state that controlled DPS under the optimal spatially continuous feedback input has the same eigenfunctions as the uncontrolled DPS, preserving its natural properties.[1,2] Therefore, the optimal control [Eqs. (19–21)] has also been referred to as "natural control."[2]

Specifically, if the desired closed-loop eigenvalues are given as the set (18) from Eqs. (19) and (20), we compute $g_{r1}^* = g_{r2}^* = 0$ for $r = n+1,\ldots,\infty$, which yields $f_r^* = 0$, $r = n+1, n+2,\ldots,\infty$.

Upon substituting this result into Eq. (21), we get

$$f^*(p,t) = \int \{ G_{1n}^*(p,p')u(p',t) + G_{2n}^*(p,p')\dot{u}(p',t)\}\, dD(p') \tag{23}$$

where G_{1n}^* and G_{2n}^* are the same as Eq. (22), except that the summation ends at n. The point is that the solution (23) does not represent a model truncation; instead, the required summations end at n because the remaining terms have been computed to be zero.

The preceding procedure indicated that optimal control can be found with virtually no effort for any eigenvalue set for the ∞-D DPS. It remains to check the control spillover effect over the uncontrolled modes $r = n+1,\ldots$. To this end, we substitute the form of $f^*(p,t)$ given by Eq. (23) into Eq. (7), which yields upon recognizing the first orthogonality in Eq. (3),

$$f_s(t) = 0, \qquad s = n+1,\ldots\infty$$

Perfect spillover elimination from residual modes is also achieved by the optimal control [Eq. (23)]. We must point out that the spillover inputs $f_s(t)$ vanish due to appearance of $\phi_r(p)$ in $f(p,t)$ regardless of the functional form of the modal control inputs $f_r(t)$, $r = 1, 2,\ldots, n$, i.e., whether modal inputs are independent or not. Therefore, spillover elimination is ultimately not a matter of what the temporal behavior of the control inputs is, but is a matter of spatial distribution of control. Any other spatial distribution of input would not yield perfect spillover elimination, at least theoretically.

Last but not the least important feature of the optimal control is that the solution is unique[1,2]; therefore, it is globally optimal, and it controls the ∞-D DPS, accomplishing the control objectives ideally.

The global control power for the optimal control for the DPS can be evaluated by substituting Eq. (21) or (23) into Eq. (8)

$$S^* = \int m^{-1} f^{*T}(p,t) f^*(p,t)\, dD \tag{24}$$

which yields from Eq. (12)

$$S^{*R} = S_\infty^{M*} = S^* \tag{25}$$

V. Suboptimal Control for the Distributed Parameter Structure

By definition, any control input field $f(p,t)$ of the form

$$f(p,t) \neq f^*(p,t)$$

will have a higher real control power than S^*. The most common suboptimal control is the one that seems most practical to implement; it is the point (or localized) input distribution

$$f(p,t) = \sum_{k=1}^{m} \delta(p - \Delta p_k) F_k(t) \tag{26}$$

where δ is the spatial Dirac delta function, Δp_k is the domain of the influence of the local kth input, and $F_k(t)$ is the total input over Δp_k.

The real control power for the suboptimal control is again computed by using Eq. (26) in Eq. (9). Denoting the total input vector by $F = [F_1\ F_2\ \ldots\ F_m]^T$, it can be shown that[5]

$$S^R = \int F(t)^T R F(t)\, dt, \qquad R = \mathrm{diag}[m(p_k)\Delta p_k]^{-1} \tag{27}$$

and from identity (12) the total modal control power corresponding to suboptimal control profile [Eq. (26)] is

$$S_\infty^M = \int_0^\infty F^T R F\, dt$$

Furthermore, the design model control cost for $\{n\}$ pairs of relocated eigenvalues is

$$S_C^M = \sum_{r=1}^{n} \int f_r^2(t)\, dt = \int f_n^T(t) f_n(t)\, dt = \int F^T B_n^T B_n F\, dt \tag{28}$$

where $f_n(t)$ is the n-component design model modal input vector generated by the control (26), and

$$f_n(t) = B_n F, \qquad B_{nrk} = [\phi_{rk}(p_k)], \qquad r = 1, 2,\ldots, n$$

where $k = 1, 2,\ldots, m$, and $\phi_{rk}(p_k)$ is the area under ϕ_r over Δp_k.

The control power spillover due to localized inputs F can be evaluated as

$$S_U^M = \sum_{r=n+1}^{\infty} \int f_r^2(t)\, dt = \int f_u^T(t) f_u(t)\, dt = \int F^T B_u^T B_u F\, dt \tag{29}$$

where

$$f_u(t) = B_u F, \qquad B_{u_{rk}} = [\phi_{rk}(p_k)], \qquad r = n+1,\ldots, \quad k = 1, 2,\ldots, m$$

and B_n and B_u are the respective partitions of the B matrix for the nth-order control design model and the uncontrolled dynamics.

One does not need to compute the control spillover performance according to Eq. (29) if S^R and S_C^M are already available, for from Eqs. (27) and (28),

$$S_U^M = S^R - S_C^M = \int F^T [R - B_n^T B_n] F\, dt \tag{30}$$

in which $F(t)$ is directly available from the control design model once it is selected. Equation (30) indicates that for a given DPS (hence the mass distribution and therefore the matrix R are known) when a control design model is selected, the control spillover performance can be determined solely on the basis of the control design model. No knowledge of the uncontrolled dynamics is needed. This points out the usefulness of the judicious definition of the control evaluation functional S in the form of Eq. (9).

The control spillover power for the suboptimal point input profile (26) cannot vanish; therefore, from Eq. (15) we deduce

$$S^R > S_C^M$$

and since S^R as given by Eq. (27) is suboptimal,

$$S^R > S^*$$

VI. Control Powers for Discrete Systems

Quite often, instead of partial differential equations, a large set of spatially discretized ordinary differential equations is assumed to describe the dynamics of the DPS adequately, such as the finite-element models (FEM) of complex structural systems. Extension of the previous definitions and results to such cases will be useful. We assume that the structural system is described by

$$M\ddot{q}(t) + Kq(t) = Q(t) \tag{31}$$

instead of Eq. (1), where $q(t)$ and $Q(t)$ are each N-component generalized coordinates and the generalized forces vectors, respectively; and M and K are $N \times N$ symmetric, positive definite mass and stiffness matrices, respectively. Equation (31) is usually referred to as the N-dimensional evaluation model replacing the ∞-D DPS [Eq. (1)].

Denoting by E the modal matrix associated with system (31), the modal transformations and the orthogonality relations

$$q = E\xi, \quad f(t) = E^T Q(t), \quad E^T M E = I, \quad E^T K E = [\omega^2]_N \tag{32}$$

again yield the modal equation of motion [Eq. (5)], with the exception that this time $r = 1, 2, \ldots, N$ represents the complete system. Again, by definition the total modal control power is

$$S_N^M = \sum_{r=1}^{N} \int f_r^2(t)\,\mathrm{d}t = \int f^T(t) f(t)\,\mathrm{d}t \tag{33}$$

where N denotes the total system as opposed to ∞ in Eq. (13) for the DPS equations [Eq. (1)]. The corresponding real control power, after recognizing the invariance property (12), is

$$S^R = S_N^M = \int f^T f\,\mathrm{d}t = \int Q^T E E^T Q\,\mathrm{d}t \tag{34}$$

Noting from the orthogonality relations that $EE^T = M^{-1}$, for a discrete system the equivalent of S^R in Eq. (9) is

$$S^R = \int Q^T M^{-1} Q\,\mathrm{d}t \tag{35}$$

Hence, the generalized input vector $Q(t)$ plays the role of $f(p,t)$.

In particular, if Eq. (31) represents the FEM equations of motion, the generalized loads vector $Q(t)$ is the vector of joint loadings. If $F(t) = [F_1\, F_2 \ldots F_N]^T$ is the real joint inputs vector of $f(p,t)$, one can write

$$Q = DF \tag{36}$$

where D is the joint loads distribution matrix. If $f(p,t)$ is spatially continuous over the whole structural domain, then by necessity it will yield an input at each joint along every joint degree of freedom. It follows that the equivalent of a spatially continuous input $f(p,t)$ in a FEM setting is tantamount to having a full generalized loads vector Q. On the other hand, if the input $f(p,t)$ is spatially discontinuous, such as in Eq. (26), that will be tantamount to having a Q with some zero components. If there are m independent joint inputs $F_1, \ldots, F_m$ as in Eq. (26), substitution of Eq. (36) into Eq. (34) yields

$$S^R = \int F(t)^T R F(t)\,\mathrm{d}t, \qquad R = D^T M^{-1} D \tag{37}$$

Hence, the weighting matrix R in Eq. (37) is the FEM equivalent of the weighting matrix R in Eq. (27) for the partial differential equations of motion.

It is clear from the form of R that, given any $F(t)$, S^R describes the control power for the entire evaluation model. Even if $F(t)$ may have been designed by considering only a reduced $\{n\}$th-order modal model of the Nth-order system (31), when S^R is computed, it will be, according to Eq. (37), the global control power for the total Nth-order evaluation model, not for the nth order reduced-control design model.

For an nth-order control design model, it is easy to see that the counterparts of Eqs. (13) and (27–29) are

$$S_N^M = S_C^M + S_U^M \tag{38}$$

where

$$S_C^M = \int F^T D^T E_n E_n^T D F\,\mathrm{d}t, \qquad S_U^M = \int F^T D^T E_u E_u^T D F\,\mathrm{d}t \tag{39}$$

$$S_U^M = S^R - S_C^M = \int F^T D^T [M^{-1} - E_n E_n^T] D F\,\mathrm{d}t \tag{40}$$

and E_n and E_u are the control design model and the uncontrolled model modal matrices, respectively. Here again, from Eq. (40), for a given physical system (hence the evaluation model mass matrix M is known) and the nth-order control design model, control spillover performance can be ascertained solely on the basis of the control design model. Specifically, if a FEM is used, the modes that are poorly computed will be inconsequential from the control point of view as long as those modes are in the uncontrolled set.

Finally, it remains to ascertain the counterparts of the globally optimal control and the control power [Eqs. (23–25)] for the system of Eq. (31). From the previous discussions and the nature of the globally optimal control, it is straightforward to obtain

$$S^* = \int Q^{*T} M^{-1} Q^*\,\mathrm{d}t = \int F^{*T} D^{*T} M^{-1} D^* F^*\,\mathrm{d}t \tag{41}$$

$$Q^* = MEf^*(t) = D^* F^*, \qquad F^* = D^{*-1} MEf^*(t) \tag{42}$$

where $f^*(t)$ is the vector of N-independent modal inputs f_r^*, $r = 1, 2, \ldots, N$, precisely as computed according to Eqs. (19), (20), or (23) if n modes are controlled, in which case the corresponding elements of f^* in Eq. (42) will be zero. For an FEM, Q^* is the full generalized inputs vector tantamount to having N joint inputs $F_r^* = 1, 2, \ldots, N$, that is, as many inputs as the total number of degrees of freedom. In other words, D^* for the globally optimal control must be a full rank N joint loads distribution matrix.

The forms of Q^* and its interpretation makes it clear that an FEM not having inputs at each joint along each joint degree of freedom would correspond to a discontinuous input profile and have a suboptimal performance.

VII. Efficiency of a Structure-Control System

Implementable control designs for large flexible structures such as complex trusslike configurations planned for the space station will inevitably employ spatially discontinuous input profiles consisting of a large number of distributed point force and torque actuators. In view of the control objectives and the features of the control powers we discussed heretofore, it would be desirable for any implementable structural control system to channel as much of the real control power as possible to the control design model. In other words, the spillover power S_U^M should be minimized by the control design. An equally desirable feature of the control design would be to keep the total control power as small as possible, that is, to keep the real control power of the design as near to the globally optimal control power as possible. These aspects, by necessity, bring about the concept of efficient structure-control designs. An important element of the structure-control design process must be to find the most efficient structure-control combination for the control objectives.

We define the percent global efficiency of a structure-control system as

$$e^*\% = \frac{S^*}{S^R} \times 100 \leq 100\% \tag{43}$$

where S^R is the real control power of any suboptimal control design with the closed-loop eigenvalue spectrum $\{\rho\} = \{\bar{\rho}\}$, and S^* is the globally optimal control power corresponding to the same eigenvalues $\{\bar{\rho}\}$. In other words, the global efficiency is based on the comparison of dynamically similar[1] globally optimal and suboptimal control designs for the desired closed-loop eigenvalues. Since S^R is suboptimal, the upper bound of global efficiency is 100%.

Next, we define the percent relative model efficiency of a structure-control system to be

$$e\% = \frac{S_C^M}{S^R} \times 100 \leq 100\% \tag{44}$$

The relative model efficiency is an indicator of the percentage of the real control power channeled into the control design model, with the balance indicating the control power spillover. This efficiency is determined solely by using the properties of the particular control design model. There is no reference to the corresponding globally minimum control. Therefore, we refer to e as the relative model efficiency. A less than perfect model efficiency automatically implies control power wasted to uncontrolled modes. However, as a 100% e means no control spillover, it will not guarantee a 100% global efficiency, since S^R and S^* may still be different because of the implied differences in their input configurations and the method of control design.

The relative model efficiency and the global efficiency are related by

$$e = \mu e^*, \qquad \mu = S_C^M / S^* \tag{45}$$

where μ is the modal efficiency coefficient.

Complementary to the preceding definitions, one can also introduce the global and the model spillover quotients

$$sq^* = S_U^M / S^* = (S^R - S_C^M)/S^* = 1/e^* - \mu$$

$$sq = S_U^M / S^R = 1 - e < 1 \tag{46}$$

where sq indicates the portion of the real control power lost as control spillover power, and sq^* indicates the control power spillover of the suboptimal design as a fraction of the globally minimum control power that would be expended on the entire DPS. Studies show that the control power spillover a suboptimal control profile can incur can be many times more than it would take to control the entire system with a spatially continuous optimal input.

Given an initial disturbance for a stable control system,

$$w_0 = [\xi_1(0)\ \eta_1(0) \ldots \xi_n(0)\ \eta_n(0)], \qquad \dot{\xi}_r = \omega_r \eta_r$$

The control costs for infinite time control are given by

$$S^R = w_0^T P^R w_0, \qquad S_C^M = w_0^T P_C^M w_0, \qquad S^* = w_0^T P^* w_0 \tag{47}$$

where P^R, P^M, and P^* are the real, modal, and the globally optimal (natural) control power matrices. P^R and P_C^M can be obtained as the solutions of the associated Lyapunov equations for any suboptimal control design discussed in the preceding sections. The natural control cost matrix P^* is given in closed form in Refs. 6 and 10.

The global and relative model efficiencies of any control design can now be computed by using the power matrices

$$e^* = w_0^T P^* w_0 / w_0^T P^R w_0, \qquad e = w_0^T P_C^M w_0 / w_0^T P^R w_0$$

$$\mu = w_0^T P_C^M w_0 / w_0^T P^* w_0 \tag{48}$$

Each one of the efficiencies, through the power matrices, depends on 1) the number, type, and locations of localized inputs $F_k(t)$; 2) the particular control design technique used to compute the actual spatially discontinuous feedback input F; 3) the order n of the control design model and the closed-loop eigenvalue spectrum $\{\bar{\rho}\}$; 4) structural parameters through the appearance of modal frequencies and mode shapes; and 5) the initial modal disturbance state w_0.

For the analysis and design of structure-control systems via efficiencies, one would typically take the following steps.

1) For any set of selected system variables and parameters mentioned earlier (such as a given nth-order model and a control input configuration), obtain a control law by whatever technique or theory deemed appropriate.

2) For the control inputs obtained in step 1, compute the corresponding S^R and S_C^M defined by Eqs. (27) and (28) or Eqs. (37) and (39) and calculate the relative model efficiency e and the model spillover quotient sq as defined by Eqs. (44) and (46), respectively.

3) For different values and/or sets of variables and parameters, repeat steps 1 and 2, compare the corresponding model efficiencies, simulate if necessary, and identify satisfactory designs consistent with the designer's criteria and constraints.

In applying steps 1–3, one should recognize that there is no need for explicit knowledge of the closed-loop eigenvalues if studies based on relative model efficiencies are all that is desired. However, in addition, if global efficiencies e^* and global spillover quotients sq^* are also desired for further consideration, one must then proceed with the following steps.

4) For the controls designed in step 1, compute the corresponding closed-loop eigenvalues $\{\bar{\rho}\}$ if they are not already available. Otherwise, this step is not needed.

5) For the spectrum $\{\bar{\rho}\}$ found in step 4, compute the modal control gains and the modal inputs given by Eqs. (19) and (20) of the globally optimal spatially continuous control, which is dynamically similar to the control design of step 1.

6) In accordance with Eqs. (25), (11), and (12), obtain the globally minimum real control cost S^* possible for the eigenvalue spectrum $\{\bar{\rho}\}$ elicited by the control design of step 1. A closed-form solution for S^* is given in Refs. 6 and 10 for any defined $\{\bar{\rho}\}$.

7) Calculate the e^* and sq^* of the control design of step 1, as defined by Eqs. (43) and (46) by using S^R from step 2 and S^* from step 6. If desired, compute the modal efficiency coefficient defined in Eq. (45).

8) For different values and/or sets of variables and parameters, repeat steps 4–6, compare the corresponding global efficiencies and spillover quotients, simulate if necessary, and identify satisfactory designs.

9) Study the results of steps 3–8 collectively to evaluate the control designs.

The efficiency approach to structure control will liberate the engineer from the need of detailed knowledge of the unmodeled modes, since the behavior of an ∞-D system can be studied and understood by means of its efficiencies, the computations of which require explicit knowledge of only the finite number of modeled control modes. This feature should make the efficiency approach to control design a practical tool.

With such an approach, it is possible to determine the optimal control input distribution and even the optimum eigenvalue distribution for a given nth-order control design model. For a given input field, the efficiencies can be used to determine the order n of a control design where model orders that yield high efficiencies can be selected. In addition, because the efficiencies are dependent on the input distribution, closed-loop eigenvalues, and other structural and control parameters, different order control models could become more efficient simply by changing the structure-control system's configuration and parameters. In all of these, the objective then should be to maximize the global and relative efficiencies.

In particular, for Nth-order discrete evaluation models, S_n^M represents the control power consumed in controlling $n < N$ modes, and S^R represents the power consumed by all N modes. Therefore, the relative efficiency e becomes a valid nondimensional measure of the effects of model order reduction. A similar statement holds true for global efficiency. Based on these observations, a closed-loop efficient model reduction technique can be formulated in that we propose to retain in the control design model the modes to which the relative model and/or global efficiencies are most sensitive for any given input configuration.

In contrast to design, efficiencies can be used to evaluate the merits of a given control design since they reflect the effects of many variables of the control problem. In this paper, we shall illustrate the analyses of some LQR control designs based on the efficiency concept. Also, the proposed concept of efficient

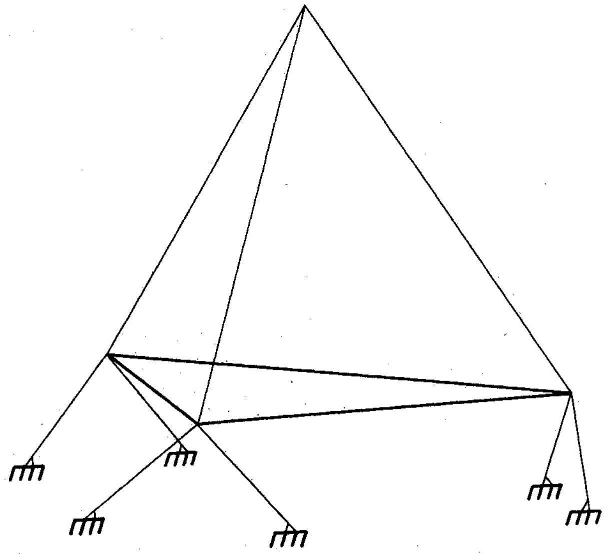

Fig. 1 ACOSS-4 tetrahedral structure.

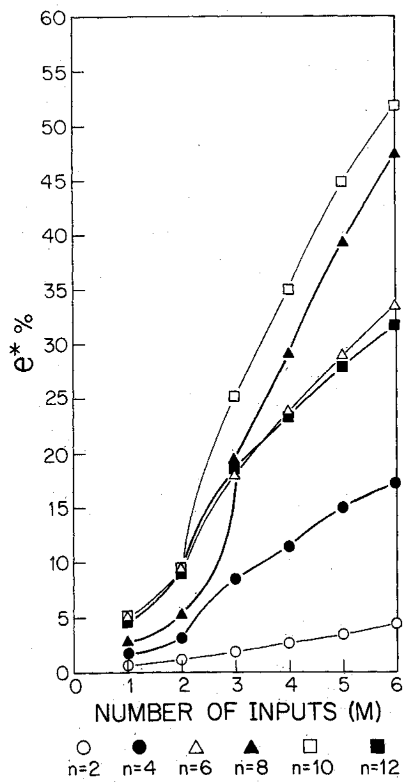

Fig. 2a Percent global efficiencies for ACOSS-4.

model reduction technique will be demonstrated. More recent theoretical developments resulting in recognition of controller efficiency modes, in addition to the familiar structural modes, and their significance for the structure-control systems, are presented in Refs. 10 and 11. The design studies via maximization of efficiencies will be left as a future endeavor.

VIII. Illustrative Example: Control of ACOSS-4 Tetrahedral Truss Structure

Analysis via Efficiencies

As a demonstration of the use of the efficiency concept, the performances of various LQR control designs (step 1) for the ACOSS-4 structure shown in Fig. 1 were evaluated for different order modal control design models and different number of inputs (steps 3 and 8). The inputs were located at the pods of the structure. A 12th-order ($N = 12$) evaluation model [Eq. (31)], obtained via FEM, was considered. For a given nth-order control design model and number of inputs $1 \leq m \leq 6$, the control designs were based on the minimization of the LQR performance measure (step 1):

$$J = \frac{1}{2}\int_0^\infty \{ w^T q[1] w + F^T r[1] F \} \, dt, \qquad q,r > 0$$

where q and r are state and control weighting parameters, respectively. The LQR design approach essentially is an indirect eigenvalue allocation. Instead of requiring explicit eigenvalue allocation, one can implicitly admit the desired eigenvalues $\{\bar{\rho}\}$ to be those of the LQR solutions for specific choices of q and r. For each LQR steady-state Riccati equation solution, the closed-loop eigenvalues were computed and assigned

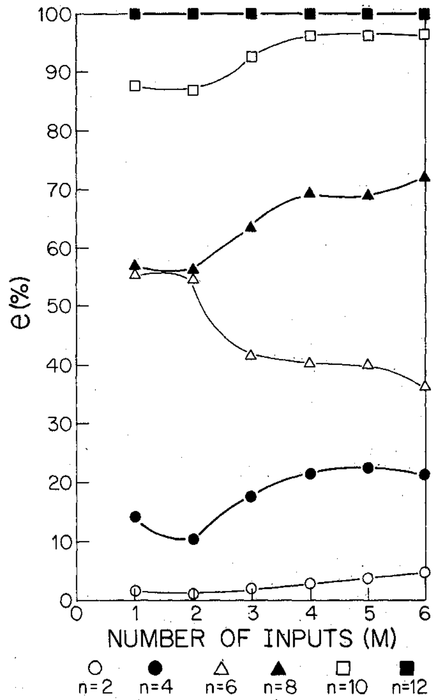

Fig. 2b Percent relative model efficiencies for ACOSS-4.

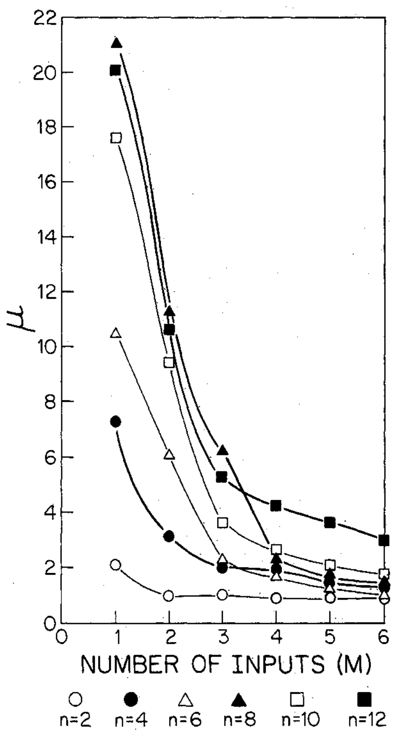

Fig. 2c Model efficiency coefficients for ACOSS-4.

Table 1 Efficiencies for ACOSS-4 with $n = 2$

m	e^*,%	e, %	μ	sq^*	sq, %
1	0.647792	1.337612	2.06488	152.3056	98.66239
2	1.271105	1.271092	0.99999	77.67170	98.72891
3	1.818071	1.859323	1.02269	53.98065	98.10468
4	2.971592	2.961611	0.99664	32.65535	97.03839
5	3.606529	3.392659	0.99615	26.73133	96.40734
6	4.642842	4.642761	0.99998	20.53855	95.35720

Table 2 Efficiencies for ACOSS-4 with $n = 8$

m	e^*,%	e, %	μ	sq^*	sq, %
1	2.70735	56.92516	21.0262	15.91033	43.07480
2	5.05710	56.82256	11.2362	8.53799	43.17744
3	19.45213	63.55967	3.2675	1.87333	36.44030
4	29.19059	69.40089	2.3775	1.04825	30.59911
5	39.41595	69.00026	1.7506	0.78648	30.99740
6	47.56904	72.06676	1.5150	0.58721	27.93324

to be the set $\{\bar{p}\}$ (step 4) where upon the corresponding globally optimal control cost S^* was computed (step 6) for the set $\{\bar{p}\}$. For each LQR solution, S^R and S_C^M were computed by solving the associated Lyapunov equations for the closed-loop system (step 2). In the simulations, the $2n$th-order initial modal state w_0 was assumed unity, and the uncontrolled modes were initially undisturbed.

For control design models of order $n = 2, 4, 6, 8, 10, 12$ and the input numbers $m = 1, 2, 3, 4, 5, 6$ (steps 3 and 8), e^*, e, μ, sq^*, and sq for LQR weighting parameters, $r = q = 1$ were computed (steps 2 and 7). The results are shown in Figs. 2 (steps 3 and 8). The model selections were made by starting from the lowest structural modes to the higher ones. For brevity, efficiencies and spillover quotients are tabulated only for $n = 2$ and $n = 8$ (Tables 1 and 2). Efficiencies of other design models and actuator configurations can be inferred from the efficiency curves.

From Figs. 2a and 2b, we observe the interactions among the efficiencies, the order of the control design model, and the input configuration. For a given number of inputs the model efficiency increases with the order of the control design model. However, this is not necessarily true for the global efficiency. For one and two inputs the global efficiency seems to increase with model order, but for three inputs increasing the model order beyond $n = 4$ decreases the global efficiency. We also observe that, for a given control design model, increasing the number of inputs increases the global efficiency, but this is not necessarily true for the model efficiency. Indeed, for a sixth-order design model ($n = 6$), three or more inputs cause a decrease in the model efficiency. Therefore, the third, fourth, fifth, and sixth actuators are located poorly with respect to the truncated modes ($n > 7$–12) such that more of control power is lost as control power spillover to cause a drop in the model efficiency. A similar observation is made for one and two inputs regarding model efficiency. Hence, it appears that the first three actuators represent a critical number of inputs for this particular structure.

The curves of sq^* and sq would describe more vividly the effect of model truncation. However, because sq^* and sq are related to e^*, μ, and e, for brevity these curves are not shown. Tables 1 and 2 list some values of the spillover quotients. From the sq^* values given in Table 1 one reads, for example, that for $m = 1$ and $n = 2$ the amount of control power lost to model truncation is 152 times the total control power that would be required to control the entire DPS with a spatially continuous optimal input profile (natural control). The control powers S^R, S_C^M for the LQR designs and the corresponding dynamically similar natural control powers S^* for $n = 2, 8$ are shown in Figs. 3. The cost plots show that natural control power can be significantly lower than the LQR powers. The distances among the cost curves are indicators of the global and model efficiencies and the modal efficiency coefficient.

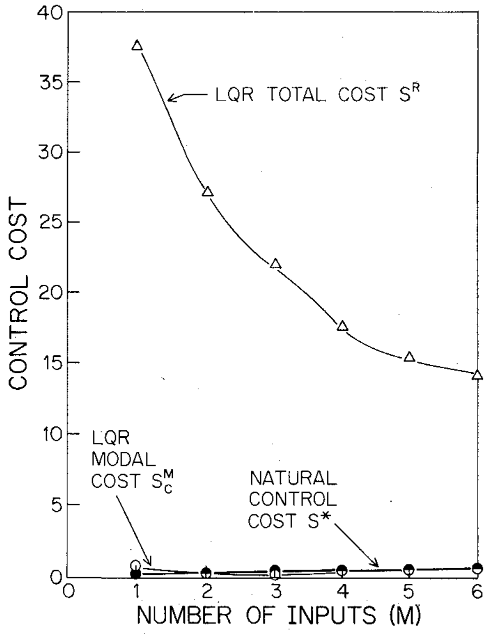

Fig. 3a Control powers for $n = 2$.

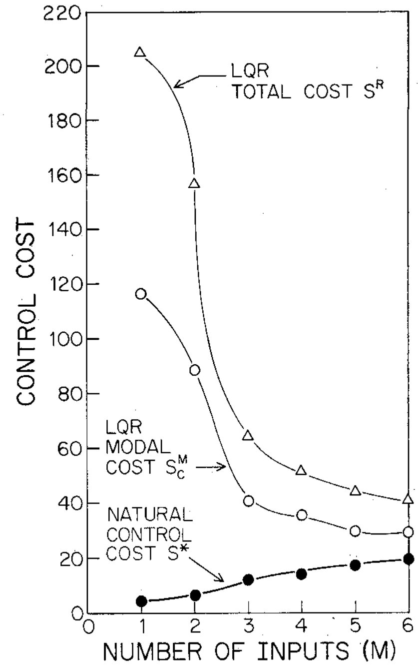

Fig. 3b Control powers for $n = 8$.

The LQR powers decrease monotonically to limit values with increasing number of inputs. The dynamically similar natural control powers increase monotonically to different limit values. We conjecture that the two designs will not converge because of fundamental differences in their design concepts. The natural control is a distributed partial differential equation control solution according to Eqs. (1), (21), and (22). On the other hand, the LQR solution is a discrete control solution based on the a priori reduced-order (truncated) model of the dynamic system. For the two optimal solutions to converge to different limits, their conceptual framework must be inherently different. This suggests that other than LQR discrete closed-loop control models can be formulated as direct approximations to the closed-loop distributed natural control solution with control powers between the LQR and natural control powers.[3]

The response profiles for a sensor colocated with the first input are given in Figs. 4 for different design model orders and

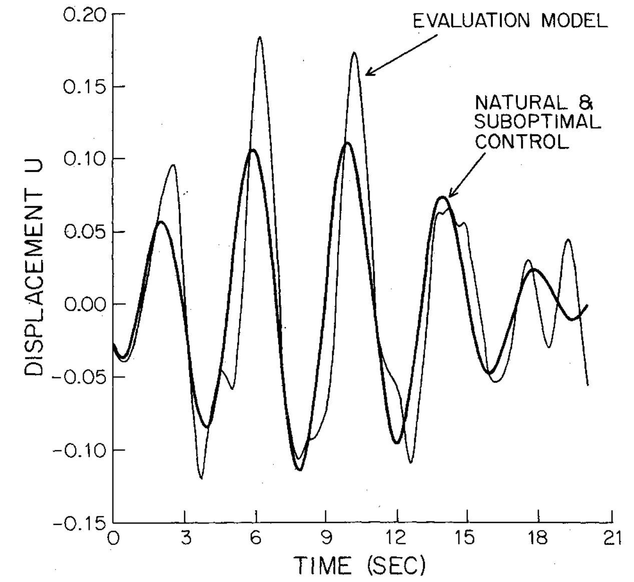

Fig. 4a Responses for $n = 2$ of natural control (continuous input) and suboptimal control and evaluation model with $m = 2$, $e^* = 1.27\%$, $e = 1.27\%$ $\mu = 1$, $sq^* = 77.67$, $sq = 98.73\%$.

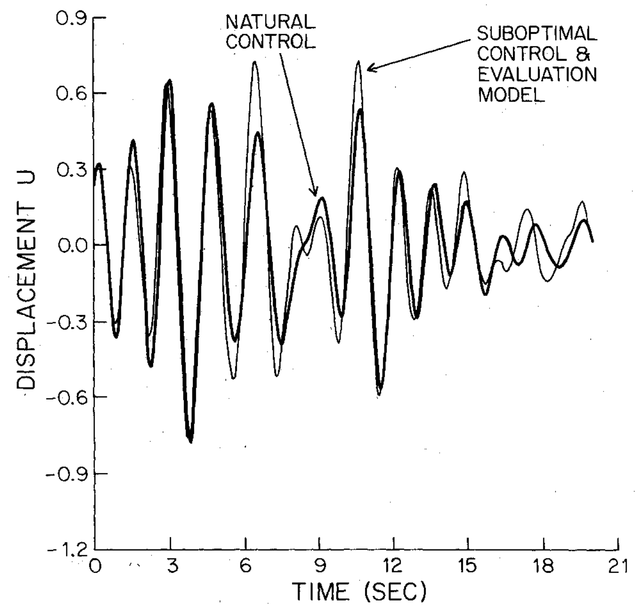

Fig. 4b Responses for $n = 8$ of natural control (continuous input) and suboptimal control and evaluation model with $m = 2$, $e^* = 5.06\%$, $e = 56.8\%$, $\mu = 11.2$, $sq^* = 8.53$, $sq = 43.2\%$.

inputs. The response profiles corresponding to the natural (globally optimal) control with continuously distributed input, suboptimal control of the nth-order control design model with m point inputs and the evaluation model, which includes the control spillover effects of the suboptimal control, are superposed in Figs. 4 for comparison purposes. It is seen that almost identical responses can be obtained with drastically different control powers. The similarity between the responses of the (suboptimal) control design model and the evaluation model for $n = 8$, $m = 2$ in Fig. 4b may suggest that model truncation is insignificant. This is true from an output viewpoint. However, there still exists a considerable inefficiency in the control design due to 43.2% control power wasted ($sq = 43.2\%$ in Table 2 for $n = 8$, $m = 2$) to truncated modes from an input viewpoint. This inefficiency can hardly be ignored. One would also want the control design to be efficient in its control power; therefore, assessment of spillover effects

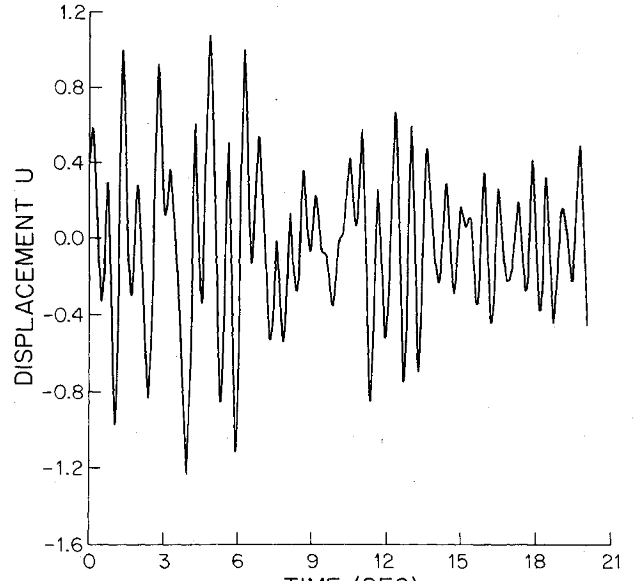

Fig. 4c Responses for $n = 12$ of natural control (continuous input) and suboptimal control with $m = 6$, $e^* = 31.8\%$, $e = 100\%$, $\mu = 3.13$, $sq = sq^* = 0$.

Table 3 Efficiencies and control costs for ACOSS-4 with an eighth-order model obtained via efficient model reduction approach

Control design modes	m	e^*,%	e, %	μ	sq^*	sq, %	S^R	S_C^M	S_U^M	S^*
3–6, 9–12	2	14.7	96.55	6.57	0.24	3.45	162.07	156.47	5.6	23.82
3–10	4	44.7	93.93	2.10	0.14	6.07	60.67	56.99	3.68	27.09

S are in energy units/second.

based on response alone without considering the control powers would be premature.

Figure 4c shows the response of the evaluation model both for natural control with continuously distributed inputs and LQR control with $m = 6$ point inputs. Both responses are identical. Because there is no mode truncation in the evaluation model, the relative model efficiency is 100%. On the other hand, the global efficiency is about 32%, reflecting the fact that the LQR solution with six-point inputs uses about three times more control power than if one were to use a spatially continuous input profile designed for independent control of all 12 modes. In spite of the increased control power, the LQR solution does not produce a response better than the natural control. In the LQR solution with six-point inputs for 12 modes, intermodal coupling of the controlled responses is inevitable. In this case, it is this coupling of the controlled modal responses that causes excessive use of control power without producing an improvement in the controlled response over that of natural control. This truly reflects the inefficiency of the control design model.

Efficient Model Reduction

An efficient model reduction concept would truncate the modes to which the model or global efficiency is least sensitive. We shall demonstrate the model reduction technique based on the model efficiency. A similar procedure can be based on the global efficiency. However, for brevity we do not demonstrate this alternate approach.

We use the model efficiency curves in Fig. 2b to find an eighth-order reduced model with two- and four-point inputs. For two inputs ($m = 2$) from Fig. 2b, we note that the smallest increments in the model efficiency are caused by modes 1, 2, 7, and 8. Hence, we retain modes 3–6, 9–12 as the control design model for the given two inputs. Similarly, for the four-input configuration ($m = 4$) from Fig. 2b, we note that

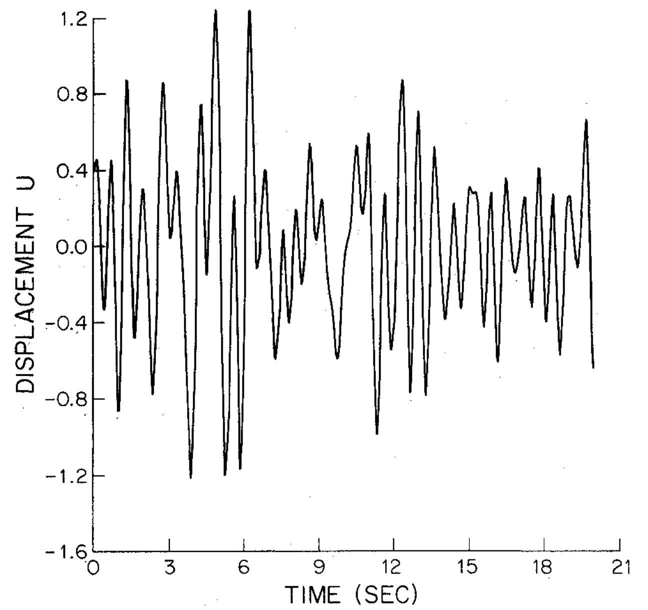

Fig. 5a Response of evaluation model with efficient model reduction applied for $n = 8$, $m = 2$, $e^* = 14.7\%$, $e = 96.58\%$, $\mu = 6.57$, $sq^* = 0.24$, $sq = 3.45\%$.

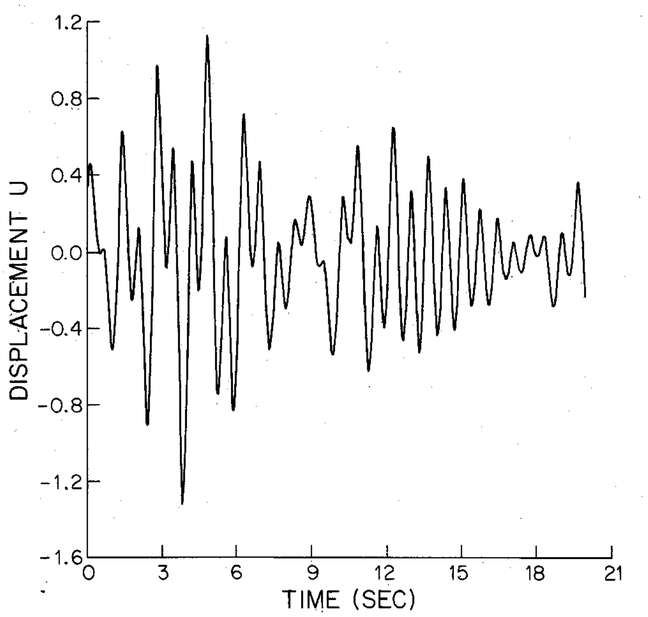

Fig. 5b Response of evaluation model with efficient model reduction applied for $n = 8$, $m = 4$, $e^* = 44.7\%$, $e = 93.93\%$, $\mu = 2.1$, $sq^* = 0.14$, $sq = 6.07\%$.

modes 1, 2, 11, and 12 have the least contributions to the model efficiency. Hence, we truncate these modes and retain modes 3–10. The efficiencies and control powers of the new eighth-order control design models for two and four inputs are shown in Table 3. A comparison of these results to the control design model, which was based on the lowest eight structural modes (Table 2), shows that the new control design models have significantly better efficiencies and the effect of model truncation for the new reduced models are insignificant. For example, for ($m = 2$), although the natural control power S^* has increased from 8 to 24 due to the mode selection based on model efficiency, the total actual control power S^R remained almost the same (158 vs 162), but the modal control power S_C^M rose from 90 to 157, which indicates that the new control design model absorbs almost all of the actual control power, yielding a 97% model efficiency, The response profiles for the new eighth-order control design models are shown in Fig. 5. Again, in these figures responses of the corresponding natural control, suboptimal control, and evaluation model are superposed. They are hardly different from each other; the responses of the suboptimal control and the evaluation model had almost undetectable overshoots at the peaks in comparison to natural control. Therefore, the curves were not labeled, and only the response of the evaluation models are shown in Figs. 5. Among all responses natural control always achieved lower amplitudes than the others. Finally, one can now compare the responses of the eighth-order reduced-order models with $m = 2$, 4 to the response of the 12th-order evaluation model in Fig. 4c with $m = 6$ inputs.

The efficient model reduction approach proposed in this paper and the controller reduction technique based on component cost analysis[7] are contrasted in Ref. 8 from a conceptual point of view.

IX. Conclusions

The concept of efficiency for structure-control systems is introduced. A global efficiency and a model efficiency can be defined for the structure-control system from the point of view of control power consumed during control. The efficiency concept is demonstrated to be a useful tool in understanding the interaction between the control variables and the structure variables and provides insight to the behavior of the structure-control system. The performance of the entire infinite-dimensional system can be studied based on the quantities obtained from the control design model alone. The efficiency concept can be used to obtain efficient reduced-order models of distributed parameter systems where for a given input configuration the modes that are most effective in increasing the efficiency of the system are retained in the design model. The paper focuses on analysis of a structure-control system. An illustrative example evaluating the performance of LQR designs for the ACOSS-4 structure has been given.

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